CADERNO DE RESUMOS

Edição especial do Workshop de Teses e Dissertações em Matemática

ICMC - USP

Sobre

Workshop de Teses e Dissertações em Matemática

Esta edição especial do **Workshop de Teses e Dissertações em Matemática** dá continuidade à proposta das edições anteriores no que diz respeito à divulgação dos trabalhos desenvolvidos pelos alunos do programa de pós-graduação em Matemática do ICMC, bem como à integração da comunidade discente do dito programa. O momento é oportuno para professores e alunos e expressa o comprometimento do ICMC com o ensino e a pesquisa em Matemática a nível de pós-graduação.

O Workshop de Teses e Dissertações do programa de pós graduação em Matemática do ICMC acontece anualmente no mês de setembro. Excepcionalmente, mediante solicitação à CCP-MAT (Comissão Coordenadora do Programa de Pós-Graduação em Matemática), é possível que o aluno realize esta etapa da avaliação durante o Programa de verão do ICMC, em edições especiais deste evento, como esta do ano de 2025.

Organização

Alan França
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Apresentações de Doutorado

GEOMETRY OF MIXED ANALYTIC VARIETIES

INÁCIO AUGUSTO RABELO PINTO,
MARIA APARECIDA SOARES RUAS, JOSÉ ANTONIO SEADE KURI

Mixed maps are real analytic maps on complex variables and their conjugates. They generalize the complex analogues and inherit several properties from this context. The central theme of the thesis project is the geometry of varieties defined by mixed maps, called mixed varieties. Our objective is twofold. Firstly, we derive conditions for topological and bi-Lipschitz equivalences of mixed singularities and associated surfaces of Pham-Brieskorn type. Secondly, we introduce the notions of Newton non-degeneracy for mixed maps and mixed isolated complete intersection singularities. For these maps, we investigate the existence of contact structures and open books on mixed links, extending a well-known property verified in the holomorphic case. Furthermore, for mixed classes related to holomorphic ones, we compare two existing contact structures and derive sufficient conditions to be isotopic.

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Apresentações de Mestrado

SINGULARITIES OF ALGEBRAIC CURVES INVARIANT BY COMPACT LIE GROUPS

AUGUSTO CEZAR ABREU RAMOS, MIRIAM GARCIA MANOEL

In this project, we investigate algebraic curves defined by in-plane polynomials that exhibit approximate symmetries. Our main objective is to correct these imperfections, seeking an approximation by an invariant curve under the action of a compact Lie group. Furthermore, we analyze situations in which these functions depend on a bifurcation parameter, leading to the formation of families of varieties that may present singularities. The study aims to understand how to adjust symmetries and describe the geometric and algebraic characteristics of the transitions between these varieties.

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STATIONARY MEASURES FOR RANDOM DYNAMICAL SYSTEMS

BELLA ROCXANE MARTINS FIGLIAGGI, TIAGO PEREIRA

This project investigates random dynamical systems and the construction of stationary measures using Markov operators. Many statistical properties, such as ergodicity and mixing, can be investigated using the Markov operator (see [1] and [2]). Stationary measures are fixed points of the Markov operator. Furthermore, the stability of convergence to these fixed points provides insights into the mixing or ergodic properties of the system (see [3]). Thus in this work we established results on the existence and stability of fixed points for this operator (that is, the existence and stability of stationary measures), as well as their stability under perturbations. Additionally, the work includes a section dedicated to iterated function systems, where Markov operator techniques are used to prove the existence and uniqueness of a stationary measure.

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GLOBAL ATTRACTORS IN HOŘAVA-LIFSHITZ COSMOLOGIES

ESTER BEATRIZ, EVERALDO BONOTTO E PHILLIPO LAPPICY

We consider spatially homogeneous models in modified gravity theories, which are expected to describe generic singularities. These models perturb the well-known Bianchi models in general relativity (GR) by a parameter $v \in (0,1)$, in which GR occurs at $v = 1/2$. We show that the well-known Rinsgtröm attractor theorem in GR extends to such modified gravity theories: generic solutions of Bianchi type IX converge to the Mixmaster attractor consisting of heteroclinic chains of Bianchi type II orbits for all $v \in (1/2, 1)$.

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AN INTRODUCTION TO RANDOM MATRICES

GABRIEL PASSARELLI, GUILHERME SILVA

Random matrices are a central topic in mathematical physics today. Although the first works in this field date back to the 1920s, which aimed to study correlation matrices of data samples, we can say that the origin of the main problems in random matrices is strongly associated with the works of physicists such as Dyson and Wigner in the 1950s and 1960s. At that time, their aim was to describe quantum mechanical systems through a probabilistic formulation, with special focus on heavy atomic nuclei. Today, however, random matrices have found applications in many areas of research, such as number theory, numerical analysis, complex networks, big data, and many more.

The definition of a random matrix model is simple to understand: we choose a set of matrices (common choices would be, for example, hermitian matrices, orthogonal matrices, unitary matrices, etc.), and over this set we define a probability distribution. Historically, one of the most studied models has been the *Gaussian Unitary Ensemble*, which deals with hermitian matrices distributed according to a probability measure that is invariant under unitary conjugation and has a Gaussian weight density. It's in this setting that the well-known Tracy-Widom distributions appear, which establish links between the theory of random matrices and the analysis of solutions of some differential equations, such as the Painlevé equations and also the Kardar-Parisi-Zhang (KPZ) stochastic equation.

In our work, we intend to give an introduction to the topic of random matrices. We'll discuss some of the main results of the field in a historical point of view, present the concept of *universality*, which underlies many models of random matrices, and we'll finish shortly discussing a tecnical result used to obtain the large limits that carachterize the universality phenomenon in the case of invariant ensembles.

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SHAPE OPTIMIZATION IN ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

KALEL BISPO GIMENEZ DE ARAUJO, EDERSON MOREIRA DOS SANTOS

Optimization problems are widely studied, for example improving some properties of an object, making its production cost cheaper, making it lighter, or more resistant. We are in particular interested in *shape optimal design problems*. Mathematically speaking, let u be the solution of a partial differential equation on a domain $\Omega \subset \mathbb{R}^N$, an *optimal shape design problem* consists in finding Ω in a class $\mathcal{O}$ of admissible domains that minimizes a given real-valued function $J(u_\Omega, \Omega)$. In our framework, we consider problems that for each admissible $\Omega \in \mathcal{O}$, it corresponds to a unique function u_Ω, that turns to be a solution of an elliptic partial differential equation posed on Ω. We also consider some suitable topologies τ for the set $\mathcal{O}$ in such a way that J is lower semicontinuous and $(\mathcal{O}, \tau)$ has good compactness properties. So to guarantee the existence of the optimal shape we need extra assumptions. Under some geometrical/regularity assumptions on the admissible domains, we prove the existence of optimal shape domains for a wide class of problems.

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